QUALITY CONTROL IN BEARING MANUFACTURING COMPANY USING STATISTICAL PROCESS CONTROL (SPC)

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the requirements for the award of the degree of
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EXAMINERS APPROVAL DOCUMENT

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We certify that the project entitled "*Quality Control in Bearing Manufacturing Company Using Statistical Process Control (SPC)*" is written by *Panirchelvan Ramanathan.* We have examined the final copy of this project and in our opinion; it is fully adequate in terms of scope and quality for the award of the degree of Bachelor of Engineering. We herewith recommend that it be accepted in partial fulfillment of the requirements for the degree of Bachelor of Mechanical Engineering with Manufacturing Engineering.

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SUPERVISOR'S DECLARATION

I hereby declare that I have read this thesis and in my opinion this thesis is sufficient in terms of scope and quality for the award of the degree of Bachelor of Mechanical Engineering with Manufacturing Engineering.

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STUDENT DECLARATION

I declare that this thesis entitled *"Quality Control in Bearing Manufacturing Company Using Statistical Process Control (SPC)"* is the result of my own research except as cited in the references. The thesis has not been accepted for my degree and is not concurrently candidature of any other degree.

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DEDICATION

To my beloved parents, Ramanathan Sockalinggam, Danalhatmi Yalumalai, and not to forget to my beloved Renuka Devi Ramachandran, my respectful brother, Tevan Ramanathan and sister, Dellagawathi Ramanathan. This thesis is the token in reciprocation of your scarifies that you made to brought me up and the strong supports.

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ABSTRACT

Statistical process control(SPC) is an important tool used widely at manufacturing field to monitor the overall operation. SPC can be applied to all kind of manufacturing operations. The significant application of the SPC analysis elements to the operation will make the process more reliable and stable. For the case study, bearing manufacturing process were selected. Bearing is one of important part in mechanical devices. A complete bearing consist of outer ring, steel ball, cage, metallic shield and inner ring. The dimensions of the bearing components are must control tight to its tolerance to ensure the bearing fit to its clearance. To ensure the process is under the production control statistical process control (SPC) methods are being used. To converge the study on SPC application on the bearing manufacturing processes, a visit to SKF Bearing Industries (Malaysia) Sdn. Bhd. has been made. The case study was done on bearing product number 6206-2Z. The bearings outer ring manufacturing processes were take into consideration. Each processes of the outer ring making were observed carefully. The quality control (QC) data were obtained from SKF and further analysis on the data was done manually and by using MINITAB software. The analyzed data were compared and the root problems and errors were identified. The problems are further discussed and some recommendations were made to make the process more efficient. In the bearing industry, bearings outer ring making operation is on of the critical process. Significant differences in dimension and deviation from the parts tolerance will make the bearing produced is rejected or categories as low quality bearing. A class 1 bearing will have tight clearance. Statistical process control will monitor the processes and magnifies the small deviations of the process from the actual control limits. Thus, the manufacturing processes can be controlled directly and good bearing which meets all the specifications can be produced.

ABSTRAK

Bearing adalah satu componen penting dalam alat-alat mekanikal yang digunakan pada masa kini. Bearing terdiri daripada gelung luar, gelung dalam, bebola besi, sangkar bebola dan penutup. Semua ukuran bahagian bearing kenala tepat. Bearing yang bahagianya tidak tepat akan ukuran tidak boleh berfungsi dengan betul dan ia boleh rosak dengan cepat. Oleh yang demikian, process pembuatan setiap bahagian bearing kenalah di beri tumpuan yang tinggi. Kawalan Process Statistikal (SPC) digunakan untuk mengukur setiap detik prosess pembuatan bearing. Untuk kajian pembelajaran, satu lawatan telah dibuat ke SKF Bearing Industries (Malaysia) Sdn. Bhd. Setiap process pembuatan bearing di SKF telah diteliti. Process pembuatan gelang luar telah dipilih untuk kajian seterusnya. Aplikasi SPC dalam pembuatan gelang luar diperhatikan. Data pengawalan qualiti(QC) telah deperolehi dari SKF. Data tersebut di kaitkan dengan applikasi SPC. Pengiraan manual dan analisis melalui perisian MINITAB telah dibuat dengan mengunakan data QC tersebut. Keputusan data di bandingkan dan setiap punca masalah dikenal pasti. Setiap masalah tersebut dibincang dengan lebih meluas. Cara-cara mengatasi masalah tersebut dicadangkan pada akhir kajian. Setiap process pembuatan bearing boleh di perhatikan dengan teliti dengan bantuan SPC. Dengan itu bearing yang berkualiti boleh dihasilkan melalui kawalan SPC yang teliti.

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LIST OF SYMBOLS

$\bar{\bar{X}}$	-	Average of the subgroup average
$\bar{X}$	-	Average of subgroup
m	-	Number of subgroups
UCL	-	Upper Control Limit
LCL	-	Lower Control Limit
σ	-	Population standard deviation of the subgroup averages
$\bar{R}$	-	Average of the range
R	-	Individual range value for the sample
A_2	-	Approximation factor used to calculate control limits
σ_R	-	Population standard deviation of the subgroup ranges
D_3	-	Approximation factor used to calculate range chart control limits
D_4	-	Approximation factor used to calculate range chart control limits
PCR	-	Process capability ratio
6s	-	6-sigma
2s	-	2-sigma
T	-	Tolerance (Specification tolerance)
d_2	-	Approximation factor for calculating within subgroup standard deviation

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

This chapter will discuss about the project background, the problem statement, objectives and scopes of the project.

1.2 PROJECT BACKGROUND

Statistical process control (SPC) involves using statistical techniques to measure and analyze the variation in processes. Most often used for manufacturing processes, the intent of SPC is to monitor product quality and maintain processes to fixed targets. Statistical quality control refers to using statistical techniques for measuring and improving the *quality* of processes and includes SPC in addition to other techniques, such as sampling plans, experimental design, variation reduction, process capability analysis, and process improvement plans.

The consistent, aggressive and committed use of SPC to bring all processes under control, recognize and eliminate special causes of variation, and identify the capability of all operations is a key requirement. SPC is defined as pr5evention of defects by applying statistical methods to control the process.

In this project the relevant SPC method applied to bearing outer ring production process at SKF Bearing Industries (Malaysia) Sdn. Bhd. The process analyzed using SPC method and its effectiveness is studied.

1.3 PROBLEM STATEMENT

Ball bearing consist of outer ring, steel ball, cage, metallic shield and inner ring. The manufacturing of each components of the bearing must be very much precise to control the quality of the bearing. The manufactured bearing in industry is classified according to its tolerance and vibration level by class 1,2 and 3. Class 1 bearing is the bearing with most perfect fitness where all of its components are exact with its dimension. Most of the customers request is on class 1 bearing where its more reliable and long lasting. In order to keep the production of the bearing components in control to its dimension; an accurate and effective process control method is required.

1.4 PROJECT OBJECTIVES

The objectives of this project are determined. There are two objectives have been defined to be focused on and to simplify the project as stated below:

i) To perform a quality control technique for a selected manufacturing process using statistical process control method.

ii) To propose method to improve the selected manufacturing process base on the case study.

1.5 SCOPE OF STUDY

The project objective is narrowed down by performing scopes of study. Firstly a comprehensive literature review has been conducted to determine the best quality statistical method. Secondly a case study has been conducted at SKF Bearing Industries (Malaysia) Sdn. Bhd. on 6206-2Z bearings outer ring production. Then, the processes of the case study analyzed using statistical process control method. Lastly the methods to improve the manufacturing processes further were proposed base on the analysis outcome.

CHAPTER 2

LITERATURE REVIEW

2.1 STATISTICAL PROCESS CONTROL (SPC) BACKGROUND

Statistical Process Control (SPC) is a control mechanism whereby measurements of product quality are actively obtained and charted simultaneously as industrial products are produced. Control is obtained when a statistical measurement such as means of a group of products are within certain control limits drawn on the statistical process chart. For these charts, there are certain set of rules to follow that will tell the technicians when a process may be out of control. When these conditions are observed, the technicians are expected to stop the manufacturing process so that corrective actions can be taken. (Dougles .C.Montgomery ,2003)

2.2 RATIONAL SUBGROUPS

Taking measurements of products on an assembly line can be costly or can potentially damage product quality. Naturally, companies often do not measure all products produced on an assembly line as this may prove to be counter-productive or economically not feasible. A compromise is often taken. For measurement of consistent product quality, most companies select what is termed as 'rational subgroups' of products. Some random sampling scheme is usually employed to secure these 'rational subgroups'. These 'rational subgroups' are then laboriously checked for quality. Measurements emanating from these 'rational subgroups' are then charted for quality control purposes. (Dougles .C.Montgomery ,2003)

2.3 APPLYING SPC IN MANUFACTURING PROCESSES

When SPC is introduced to a process, the initial thought should be about the quality characteristics. What are they and what are their associated measurement requirements? In most situations, a statistical quality control (SQC) analysis would be done first by identifying the quality characteristics are critical to quality (CTQ) and determining how those characteristics can be controlled.

As an example, when an automobile transmission or a four-wheel-drive transfer case is made, one of the quality characteristics is the noise (or lack of it) at various speeds. The SQC analysis of the noise at specific speeds could then lead to several SPC charts of CTQ measurements in the manufacturing of the gears that would contribute to the noise factor. Below are the steps in brief of existing SPC in manufacturing process. (Dougles .C.Montgomery ,2003)

i) The initial step in SPC is diagramming and analyzing the process to decide where control charts may be best applied.
ii) Decrease any obvious variability in the target process.
iii) The third step involves statistically testing the gauges using a gauge capability study. This must be done before measurements are taken for control charting. The variation that shows up on the control charts must reflect the process variation that needs to be reduced.
iv) Make a sampling plan. Determine the size of the sample and when the samples are to be taken.
v) By using control chart, find the out-of-control situation caused by common-cause and special-cause, evaluates what happened at that specific time to cause it, and then work to prevent that cause. This procedure continues until the control chart indicates that there are no more special-cause variation problems. By this time, the process is running as well as it possibly can without process modifications and it is said to be in statistical control.
vi) The sixth step is to put operator in-charged. This step and step 5 actually occurred simultaneously because the operator should be doing the control

charting and attaining statistical control with the help of the process control team.

vii) This step is to determine how capable the process is according to product specifications and customer expectations.

viii) This step is designed to improve the process. Eighty-five percent or more of the process problem are handheld at this stage, according to quality consultant W.Edwards Deming. At this stage, process changes can be analyzed on control charts either singly or in variable interaction studies for signs of process improvement. Designed experiments may also be used in the search for improvements. When improvements are found, management must follow thru and see the appropriate changes are incorporated in the process without backsliding. (S B Billatos, B S Kim)

ix) The ninth step calls for a switch to pre-control, a monitoring technique that compares a measurement with target and warning measurements, when the process is in control and capable.

x) Quality improvement is a continuous process. Two things should be done at this step; first, continue to look for ways to improve the process at hand and second, return to step 1 for the next critical measurement.

Historically, many companies did not begin using SPC until they were forced. Either they could see their competitive position diminishing or they were obliged to meet their customers requirement that contracts would not be awarded until their workforce was trained in SPC. Unfortunately, some companies just met the minimum requirement of providing basic SPC training and discovered that it’s was not good enough.

Both workers and supervisors must understand the essential of the SPC for their own good to prevent losses becoming routine. The management also must have good interpretation on SPC where when workers suggestion is made based on SPC analysis, the management should not neglect their recommendation.

2.4 ANTICIPATED BENEFITS OF IMPLEMENTATION

SPC is a powerful tool to optimize the amount of information needed for use in making management decisions (Dougles .C.Montgomery ,2003). Statistical techniques provide an understanding of the business baselines, insights for process improvements, communication of value and results of processes, and active and visible involvement. SPC provides real time analysis to establish controllable process baselines; learn, set, and dynamically improve process capabilities; and focus business on areas needing improvement. SPC moves away from opinion-based decision making. (S B Billatos, B S Kim)

These benefits of SPC cannot be obtained immediately by all organizations. SPC requires defined processes and a discipline of following them. It requires a climate in which personnel are not punished when problems are detected. It requires management commitment. (Samuel Kotz, 2002)

Statistical process control is an important tool and leads to many process improvements and positive process results, such as:

- Uniformity of output.
- Reduced rework.
- Fewer detective product.
- Increased profit.
- Lower average cost.
- Fewer errors.
- Higher quality output.
- Less scrap.
- Less machine downtime.
- Less waste in production labor hours.
- Increased job satisfaction.
- Improved competitive position.
- More jobs.

2.4.1 SPC Implementation Issues

Implementing SPC in a manufacturing process starts with defining the process its self. Consistent measurements cannot be expected from software processes that are not documented and generally followed. Then appropriate measures must be chosen. Measures need not be exhaustive. One or two measures that provide insight into the performance of a process or activity are adequate, especially if the measures are related to the process or activity goal. Measures that can be tracked inexpensively are preferable. After that, the process trends must be focused. Control charts should be constructed so as to detect process trends, not individual nonconforming events. Eventually the investigation on the chart trend must be done. Appropriate action should be taken for each causes. SPC only signals the possible existence of a problem. Without detailed investigations, as in an audit, and instituting corrective action, SPC will not provide any benefit. Finally the operating personals should provided needed training. Problems in following the above recommendations for implementing SPC can be decreased with effective training. SPC training based on examples of software processes is to be preferred.

2.5 THE BASIC TOOL FOR SPC

Usually, there are **seven commonly recognized tools** or **diagrams** for statistical process control:

1. Check sheet
2. Run chart
3. Histogram
4. Pareto chart
5. Scatter diagram/chart
6. Cause and effect or fishbone diagram
7. Control chart

Some basic examples are shown in following which we have cited from (Fred Spring, 1995) only for illustration the general characteristics.

Check Sheet: The check sheet (see Table 2.1) is used for counting and accumulating data in a general or special context.

Table 2.1: Check sheet Used for Counting and Accumulating Data

Student Name	*In Class This Week?*
Jane	√ √ √ √ √
Robert	√ √ √ √ √
Jennifer	√ √ √
Puff Daddy	

Source : Fred Spring, 1995

Run Chart: The run chart (see Figure 2.1) tracks trends over a period of time. Points are tracked in the order in which they occur. Each point represents an observation. We can often see interesting trends in the data by simply plotting data on a run chart. A danger in using run charts is that we might overreact to normal variations, but it is often useful to put our data on a run chart to get a feel for process behavior.

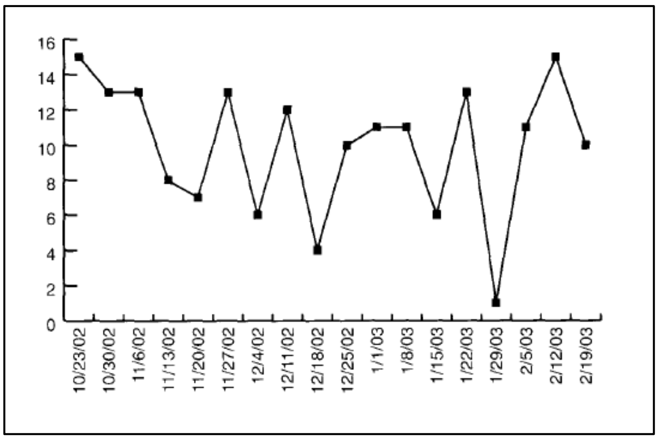

Figure 2.1: Example of a run chart

Source : Thomas, 1987

Histogram: The histogram (see Figure 2.2) is a bar chart that presents data that have been collected over a period of time, and graphically presents these data by frequency. Each bar represents the number of observations that fit within the indicated range. Histograms are useful because they can be used to see the amount of variation in a process. The data in this histogram are the same data as in the run chart in Figure 1. Using the histogram, we get a different perspective on the data. We can see how often similar values occur and get a quick idea of how the data are distributed.

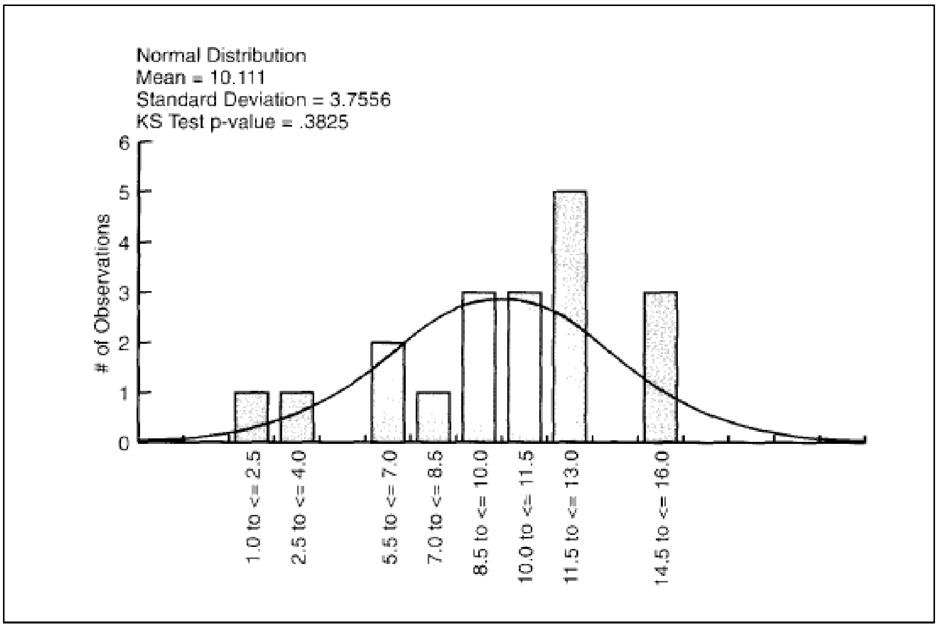

Figure 2.2: Example of a Histogram

Source : Thomas, 1987

Pareto Chart: The Pareto chart (see Figure 2.3) is a bar chart that presents data prioritized in some fashion, usually either by descending or ascending order of importance. Pareto diagrams are used to show attribute data. Attributes are qualitative data that can be counted for recording and analysis; for example, counting the number of each type of defect. Pareto charts are often used to analyze the most often occurring type of something.

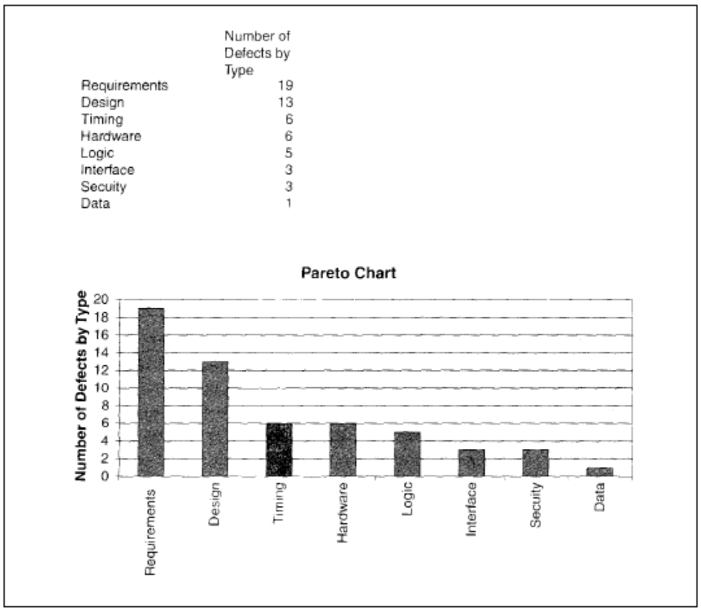

Figure 2.3: An example of a Pareto chart (Thomas, 1987)

Source : Thomas, 1987

Scatter Diagram/Chart: The scatter diagram (see Figure 2.4) is a diagram that plots data points, allowing trends to be observed between one variable and another. The scatter diagram is used to test for possible cause-and-effect relationships. A danger is that a scatter diagram does not prove the cause-and-effect relationship and can be misused. A common error in statistical analysis is seeing a relationship and concluding cause-and-effect without additional analysis.

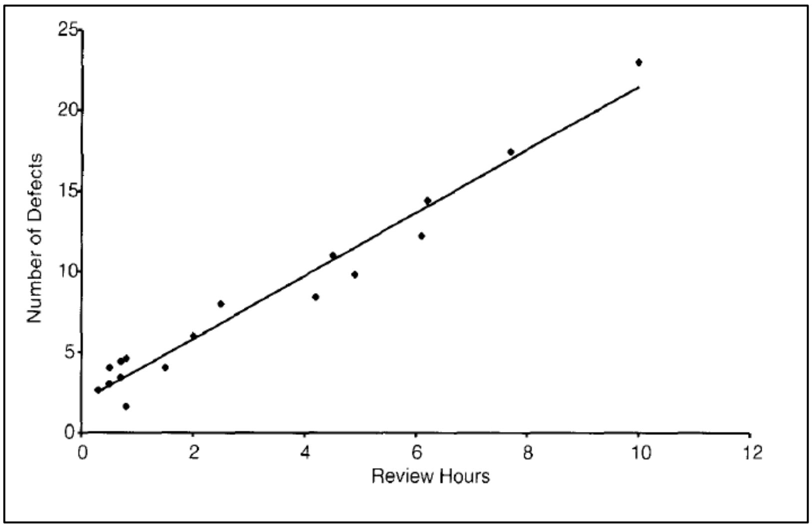

Figure 2.4: An example of a scatter diagram/chart

Source : Thomas, 1987

Cause-and-Effect/Fishbone Diagram: The cause-and-effect/fishbone diagram (see Figure 2.5) is a graphical display of problems and causes. This is a good to capture team input from a brainstorming meeting, from a set of defect data, or from a check sheet.

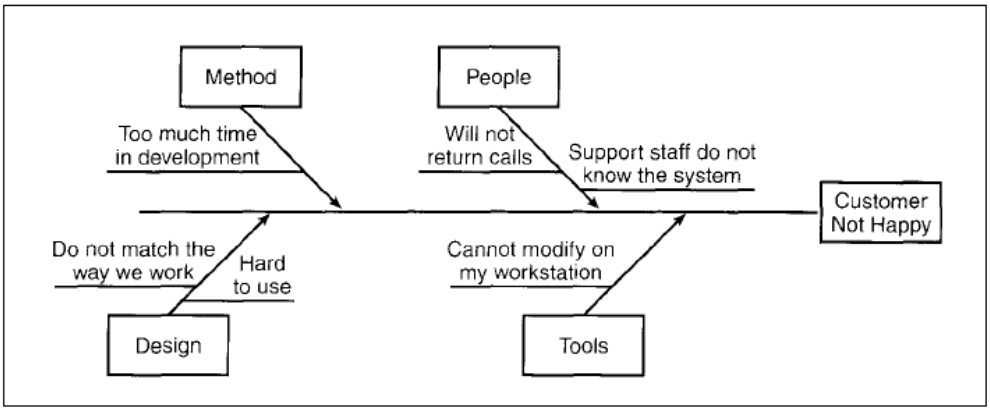

Figure 2.5: A cause and effect/fishbone diagram example

Source : Thomas, 1987

Control Chart: The control chart (see Figure 2.6) is basically a run charts with upper and lower limits that allows an organization to track process performance variation. Control charts are also called process behavior charts.

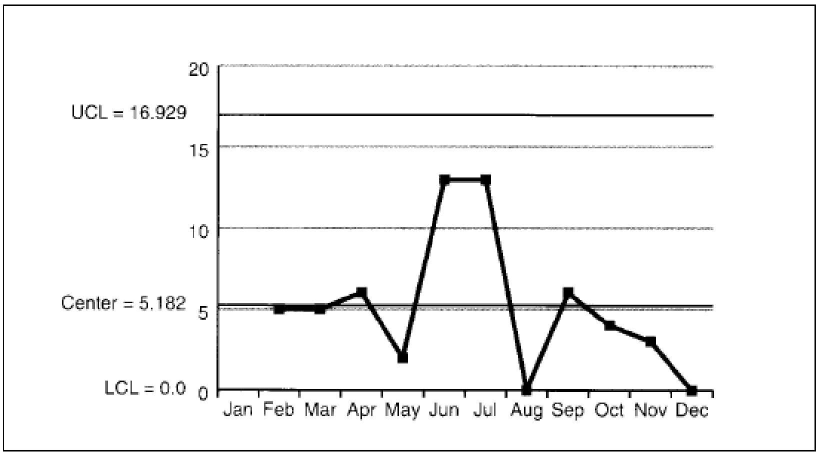

Figure 2.6: Example of a control chart

Source : Thomas, 1987

These seven graphical displays can he used together or separately to help gather data, accumulate clam, and present the data for different functions associated with SPC.

2.5.1 Example of SPC Tools Presentations in the Manufacturing Process.

Table 2.2 : SPC tools and its example of application

Tool	Example of Application
Check Sheet	To count occurrences of problems.
Run Chart	For visualize the trend of data distribution.
Histogram	To identify central tendencies and any skewing to one side or the other.
Pareto Chart	To identify the 20% of the modules which yield 80% of the issues.
Scatter Diagram	For identifying correlation and suggesting causation.
Cause and Effect Diagram	For identifying assignable causes.
Control Chart	For identifying processes that are out of control.

Source : Michael V Petrovich

2.6 VARIATION IN A MANUFACTURING PRODUCT LINE

The most important goal of understanding the principle of natural process variation is to consider the natural variance in the output before we make any changes to the process. Since SPC tends to minimize the process variations in time, as we better understand the process and have more experience with running it, we try to reduce the variation of it. The knowledge of the principle of natural variance helps us avoid making any unnecessary changes to the process, which might add variance to the process, instead of removing it. The technical goal is to reduce process variation such that the amount of unacceptable product is not more that 3 defects per million parts. (Michael V Petrovich)

Here are a few examples:

1. We manufacture tires and the tread depth needs to be 5/8 inch plus or minus 0.05 inch.
2. We approve loans and we promise a response to the customer within 24 business hours of receipt.
3. We write code and the manager expects less than 5 bugs found over the life of the product per thousand lines of code written.
4. We process invoices for healthcare services and the customers expect zero errors on their bills.

So how we can determine if the process is scattered around the target, grouped well but off the target? We can display our data in frequency distributions showing the number (percentage) of our process outputs having the indicated dimensions. (Michael V Petrovich)

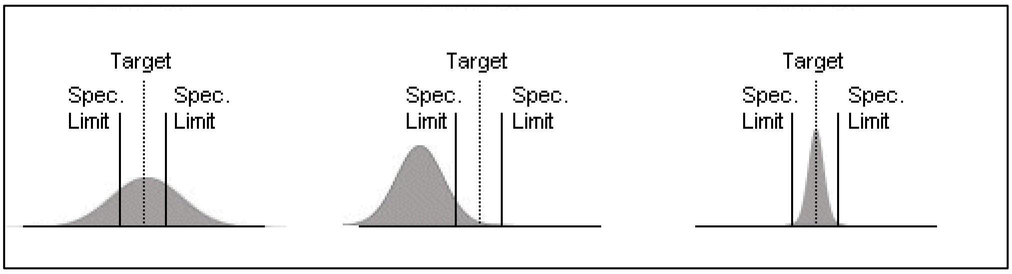

Figure 2.7 : Targeting Process Variation With The Process Capability Ratio

Source : Thomas, 1987

In Figure 2.7, the far left picture displays wide variation that is centered on the target. The middle picture shows little variation, but off target. And the far right picture displays little variation centered on the target. Shaded areas falling between the specification limits indicate process output dimensions meeting specifications; shaded areas falling either to the left of the lower specification limit or to the right of the upper specification limit indicate items falling outside specification limits. (Phillip L Ross, 1998)

2.7 PRINCIPLES OF SPC TECHNIQUES

There are a few key principles of SPC techniques. In any process or system, variation is to be expected. By use of simple statistical techniques we can define the limits of variation beyond which data points are deemed worthy of investigation. These limits are known as control limits. Variation within these limits is often called common-cause or process variation; variation outside these limits is often called special-cause or extra-process variation. Common-cause variation is that which can be expected to occur in a stable process or system - one which is ‘under control’. Special-cause variation may derive from systematic or unexpected deviation from the norm and may highlight an area or an observation which is worthy of further investigation. A useful estimate of expected ‘performance’ of a system is often the group average, and the best estimate of expected variation around the group average is ±3 standard deviations(SDs) (roughly equivalent to 99.8% confidence intervals). This degree of variation has both empirical and theoretical justification. These limits (control limits) can be readily derived and depend on the nature of the data being used to assess the process.

2.8 TYPES OF CONTROL CHARTS

Control charts are used to identify process variation over time. All processes vary. The degree of variance, and the causes of the variance, can be determined using control charting techniques. While there are many types of control charts, the ones we have seen the most often are the : (Phillip L Ross, 1998)

i) C-chart:

This chart uses a constant sample size of attribute data, where the average sample size is greater than five. It is used to chart the number of defects (such as “12” or “15” defects per thousand lines of code). c stands for the number of nonconformities within a constant sample size.

ii) U-chart:.

This chart uses a variable sample size of attribute data. This chart is used to chart the number of defects in a sample or set of samples (such as “20 out of 50” design flaws were a result of requirements errors). u stands for the number of nonconformities with varying sample sizes.

iii) np-chart:

This chart uses a constant sample size of attribute data, usually greater than or equal to 50. This chart is used to chart the number defective in a group. For example, a hardware component might be considered defective, regardless of the total number of defects in it. np stands for the number defective.

iv) p-chart:

This chart uses a variable sample size of attribute data, usually greater than or equal to 50. This chart is used to chart the fraction defective found in a group. p stands for the proportion defective.

v) X and ***mR charts***:

These charts use variable data where the sample size is one.

vi) X-bar and ***R charts:***

These charts use variable data where the sample size is small. They can also be based on a large sample size greater than or equal to ten. X-bar stands for the average of the data collected. R stands for the range (distribution) of the data collected.

vii) X-bar and ***s charts***:

These charts use variable data where the sample size is large, usually greater than or equal to ten.

For the case study purposes, **average and range (X-bar and R charts) chart** is chosen. The sample data that obtained from the visited company is suitable to use this control chart method analysis. This will be further discussed in next chapter. The features of this type of chart is discussed in the following topics.

2.9 VARIABLE CONTROL CHARTS

During the 1920's, Dr. Walter A. Shewhart proposed a general model for control charts as follows : (Wood, 1994)

2.9.1 Shewhart Control Charts for Variables

Let *w* be a sample statistic that measures some continuously varying quality characteristic of interest (e.g., thickness), and suppose that the mean of *w* is μ_w, with a standard deviation of σ_w. Then the center line, the UCL and the LCL are (Montgomery, 2005; Wood, 1994) :

$$\text{UCL} = \mu_w + k\sigma_w \quad \text{(eq 2.1)}$$

$$\text{Center Line} = \mu_w \quad \text{(eq 2.2)}$$

$$\text{LCL} = \mu_w - k\sigma_w \quad \text{(eq 2.3)}$$

Where *k* is the distance of the control limits from the center line, expressed in terms of standard deviation units. When *k* is set to 3, we speak of 3-sigma control charts. (Wood, 1994)

Historically, k = 3 has become an accepted standard in industry. (D Saravan, 2002; Constantian Anghel, 2001)

The centerline is the process mean, which in general is unknown. We replace it with a *target* or the *average* of all the data. The quantity that we plot is the sample average, $\overline{\mathbf{X}}$. The chart is called the $\overline{\mathbf{X}}$chart.

We also have to deal with the fact that σ is, in general, unknown. Here we replace σ_w with a given standard value, or we estimate it by a function of the *average standard deviation*. This is obtained by averaging the individual standard deviations that we calculated from each of *m* preliminary (or present) samples, each of size *n.* It is equally important to examine the standard deviations in ascertaining whether the process is in control. There is, unfortunately, a slight problem involved when we work with the usual estimator of σ. The following discussion will illustrate this.

2.9.2 Differences between control limits and specification limits

Control Limits are used to determine if the process is in a state of statistical control (i.e., is producing consistent output). Specification Limits are used to determine if the product will function in the intended fashion.

2.10 NUMBER OF DATA POINTS NEEDED TO SET UP A CONTROL CHART

Shewhart gave the following rule of thumb:

"It has also been observed that a person would seldom if ever be justified in concluding that a state of statistical control of a given repetitive operation or production process has been reached until he had obtained, under presumably the same essential conditions, a sequence of not less than twenty five samples of size four that are in control." (Constantian Anghel, 2001)

It is important to note that control chart properties, such as false alarm probabilities, are generally given under the assumption that the parameters, such as μ and σ, are known. When the control limits are not computed from a large amount of data, the actual properties might be quite different from what is assumed (Constantian Anghel, 2001).

2.11 SHEWHART X-BAR AND R CONTROL CHARTS

If the sample size is relatively small (say equal to or less than 10), we can use the range instead of the standard deviation of a sample to construct control charts on $\bar{X}$ and *the range, R*. The range of a sample is simply the difference between the largest and smallest observation.

There is a statistical relationship (Chin-Chuan Wu, 2004) between the mean range for data from a normal distribution and σ, the standard deviation of that distribution. This relationship depends only on the sample size, n. The mean of R is d_2 σ, where the value of d_2 is also a function of n. *An estimator of* σ is therefore R /d_2.

Armed with this background we can now develop the $\overline{\mathrm{X}}$and R control chart. Let R_1, R_2, ..., R_k, be the range of k samples. The average range is :

$$\bar{R} = \frac{R_1 + R_2 + ... + R_k}{k} \qquad \text{(eq 2.4)}$$

Then an estimate of σ can be computed as :

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \qquad \text{(eq 2.5)}$$

2.11.1 $\overline{\mathrm{X}}$control charts

So, if we use $\bar{\bar{x}}$ (or a given target) as an estimator of μ and $\bar{R}/d_2$ as an estimator of σ, then the parameters of the $\overline{\mathrm{X}}$ chart are :

$$UCL = \bar{\bar{x}} + \frac{3}{d_2\sqrt{n}}\bar{R} \qquad \text{(eq 2.6)}$$

$$\text{Center Line} = \bar{\bar{x}} \qquad \text{(eq 2.7)}$$

$$LCL = \bar{\bar{x}} - \frac{3}{d_2\sqrt{n}}\bar{R} \qquad \text{(eq 2.8)}$$

The simplest way to describe the limits is to define the factor $A_2 = 3/(d_2\sqrt{n})$ and the construction of the $\overline{\mathrm{X}}$ becomes

$$UCL = \bar{\bar{x}} + A_2\bar{R} \qquad \text{(eq 2.9)}$$

$$\text{Center Line , } \bar{\bar{X}} = \frac{\sum_{i=1}^{m} \bar{x}_i}{m} \qquad \text{(eq 2.10)}$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} \qquad \text{(eq 2.11)}$$

The factor A_2 depends only on n, and is tabled below.

2.11.2 The R Control Chart

This chart controls the process variability since the sample range is related to the process standard deviation. *The center line of the R chart is the average range.* To compute the control limits we need an estimate of the true, but unknown standard deviation $W = R/\boldsymbol{\sigma}$. This can be found from the distribution of $W = R/\boldsymbol{\sigma}$ (assuming that the items that we measure follow a normal distribution). The standard deviation of W is d_3, and is a known function of the sample size, n. It is tabulated as statistical quality control. Therefore since $R = W\boldsymbol{\sigma}$, the standard deviation of R is $\boldsymbol{\sigma}_R = d_3\,\boldsymbol{\sigma}$. But since the true $\boldsymbol{\sigma}$ is unknown, we may estimate $\boldsymbol{\sigma}_R$ by

$$\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2} \qquad \text{(eq 2.12)}$$

As a result, the parameters of the R chart with the customary 3-sigma control limits are :

$$UCL = \bar{R} + 3\hat{\sigma}_R = \bar{R} + 3d_3\frac{\bar{R}}{d_2} \qquad \text{(eq 2.13)}$$

$$\text{Center Line} = \bar{R} \qquad \text{(eq 2.14)}$$

$$LCL = \bar{R} - 3\hat{\sigma}_R = \bar{R} - 3d_3\frac{\bar{R}}{d_2} \qquad \text{(eq 2.15)}$$

As was the case with the control chart parameters for the subgroup averages, defining another set of factors will ease the computations, namely:

$D_3 = 1 - 3\,d_3 / d_2$ and $D_4 = 1 + 3\,d_3 / d_2$.
These yields

$$UCL - \bar{R}D_4 \qquad \text{(eq 2.16)}$$

$$\text{Center Line} = \bar{R} \qquad \text{(eq 2.17)}$$

$$LCL = \bar{R}D_3 \qquad \text{(eq 2.18)}$$

The factors D_3 and D_4 depend only on n, and are tabled below.

Table 2.3 : Factors for Calculating Limits for $\overline{\mathbf{X}}$ and R Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits			Chart for Ranges: Factor for Central Line	Chart for Ranges: Factors for Control Limits					Chart for Standard Deviations: Factor for Central Line	Chart for Standard Deviations: Factors for Control Limits			
	A	A_2	A_3	d_2	d_1	D_1	D_2	D_3	D_4	c_4	B_3	B_4	B_5	B_6
2	2.121	1.880	2.659	1.128	0.853	0	3.686	0	3.267	0.7979	0	3.267	0	2.606
3	1.732	1.023	1.954	1.693	0.888	0	4.358	0	2.574	0.8862	0	2.568	0	2.276
4	1.500	0.729	1.628	2.059	0.880	0	4.698	0	2.282	0.9213	0	2.266	0	2.088
5	1.342	0.577	1.427	2.326	0.864	0	4.918	0	2.114	0.9400	0	2.089	0	1.964
6	1.225	0.483	1.287	2.534	0.848	0	5.078	0	2.004	0.9515	0.030	1.970	0.029	1.874
7	1.134	0.419	1.182	2.704	0.833	0.204	5.204	0.076	1.924	0.9594	0.118	1.882	0.113	1.806
8	1.061	0.373	1.099	2.847	0.820	0.388	5.306	0.136	1.864	0.9650	0.185	1.815	0.179	1.751
9	1.000	0.337	1.032	2.970	0.808	0.547	5.393	0.184	1.816	0.9693	0.239	1.761	0.232	1.707
10	0.949	0.308	0.975	3.078	0.797	0.687	5.469	0.223	1.777	0.9727	0.284	1.716	0.276	1.669
11	0.905	0.285	0.927	3.173	0.787	0.811	5.535	0.256	1.744	0.9754	0.321	1.679	0.313	1.637
12	0.866	0.266	0.886	3.258	0.778	0.922	5.594	0.283	1.717	0.9776	0.354	1.646	0.346	1.610
13	0.832	0.249	0.850	3.336	0.770	1.025	5.647	0.307	1.693	0.9794	0.382	1.618	0.374	1.585
14	0.802	0.235	0.817	3.407	0.763	1.118	5.696	0.328	1.672	0.9810	0.406	1.594	0.399	1.563
15	0.775	0.223	0.789	3.472	0.756	1.203	5.741	0.347	1.653	0.9823	0.428	1.572	0.421	1.544
16	0.750	0.212	0.763	3.532	0.750	1.282	5.782	0.363	1.637	0.9835	0.448	1.552	0.440	1.526
17	0.728	0.203	0.739	3.588	0.744	1.356	5.820	0.378	1.622	0.9845	0.466	1.534	0.458	1.511
18	0.707	0.194	0.718	3.640	0.739	1.424	5.856	0.391	1.608	0.9854	0.482	1.518	0.475	1.496
19	0.688	0.187	0.698	3.689	0.734	1.487	5.891	0.403	1.597	0.9862	0.497	1.503	0.490	1.483
20	0.671	0.180	0.680	3.735	0.729	1.549	5.921	0.415	1.585	0.9869	0.510	1.490	0.504	1.470

Source : Wood, 1994

In general, the range approach is quite satisfactory for sample sizes up to around 10. For larger sample sizes, using subgroup standard deviations is preferable. For small sample sizes, the relative efficiency of using the range approach as opposed to using standard deviations is shown in the following table.

Table 2.4: Efficiency of R versus S

n	Relative Efficiency
2	1.000
3	0.992
4	0.975
5	0.955
6	0.930
10	0.850

Source : Wood, 1994

A typical sample size is 4 or 5, so not much is lost by using the range for such sample sizes.

2.12 RECALCULATION OF THE CONTROL LIMITS

Since a control chart "compares" the current performance of the process characteristic to the past performance of this characteristic, changing the control limits frequently would negate any usefulness.

So, only change the control limits if did have a valid, compelling reason for doing so. Some examples of reasons:

- When we have at least 30 more data points to add to the chart and there have been no known changes to the process.
    - We will get a better estimate of the variability.
- If a major process change occurs and affects the way of our process runs.
- If a known, preventable act changes the way the tool or process would behave (power goes out, consumable is corrupted or bad quality, etc.) (Thomas, 1987)

2.13 ANALYSIS OF OUT-OF-CONTROL

2.13.1 WECO (Western Electric Company Rules) rules for signaling "Out of Control".

General rules for detecting out of control or non-random situations (Thomas, 1987)

Any Point Above +3 Sigma

-- +3 σ LIMIT

2 Out of the Last 3 Points Above +2 Sigma

-- +2 σ LIMIT

4 Out of the Last 5 Points Above +1 Sigma

-- +1 σ LIMIT

8 Consecutive Points on This Side of Control Line

=================================== **CENTER LINE**

8 Consecutive Points on This Side of Control Line

-- -1 σ LIMIT

4 Out of the Last 5 Points Below - 1 Sigma

-- -2 σ LIMIT

2 Out of the Last 3 Points Below -2 Sigma

-- -3 σ LIMIT

Any Point Below -3 Sigma

Trend Rules : 6 in a row trending up or down. 14 in a row alternating up and down.

1. WECO rules based on probabilities

The WECO rules are based on probability. We know that, for a normal distribution, the probability of encountering a point outside ± 3σ is 0.3%. This is a rare event. Therefore, if we observe a point outside the control limits, we conclude the process has shifted and is unstable. Similarly, we can identify other events that are equally rare and use them as flags for instability. The probability of observing two points out of three in a row between 2σ and 3σ and the probability of observing four points out of five in a row between 1σ and 2σ are also about 0.3%. (Chin-Chuan Wu, 2004)

2. WECO Rules Increase False Alarms

Note: While the WECO rules increase a Shewhart chart's sensitivity to trends or drifts in the mean, there is a severe downside to adding the WECO rules to an ordinary Shewhart control chart that the user should understand. When following the standard Shewhart "out of control" rule (i.e., signal if and only if we see a point beyond the plus or minus 3 sigma control limits) we will have "false alarms" every 371 points on the average. Adding the WECO rules increases the frequency of false alarms to about once in every 91.75 points, on the average. We has to decide whether this price is worth paying (some add the WECO rules, but take them "less seriously" in terms of the effort put into troubleshooting activities when out of control signals occur). (Thomas, 1987)

2.13.2 Time To Detection or Average Run Length (ARL)

Waiting time to signal "out of control"

Two important questions when dealing with control charts are:

1. How often will there be false alarms where we look for an assignable cause but nothing has changed?
2. How quickly will we detect certain kinds of systematic changes, such as mean shifts?

The ARL tells us, for a given situation, how long on the average we will plot successive control charts points before we detect a point beyond the control limits. For an $\overline{\mathbf{X}}$ chart, with no change in the process, we wait on the average $1/p$ points before a false alarm takes place, with p denoting the probability of an observation plotting outside the control limits. For a normal distribution, p = .0027 and the ARL is approximately 371.

2.14 CENTRAL LIMIT THEOREM

The central limit theorem can be stated as follows:
Irrespective of the shape of the distribution of the population or universe, the distribution of average values of samples drawn from that universe will tend toward a normal distribution as the sample size grows without bound. (AJsupF & Watson, 1993)

It can also be shown that the average of sample averages will equal the average of the universe and that the standard deviation of the averages equals the standard deviation of the universe divided by the square root of the sample size. Shewhart performed experiments that showed that small sample sizes were needed to get approximately normal distributions from even wildly non-normal universes. Figure 2.8 was created by Shewhart using samples of four measurements.

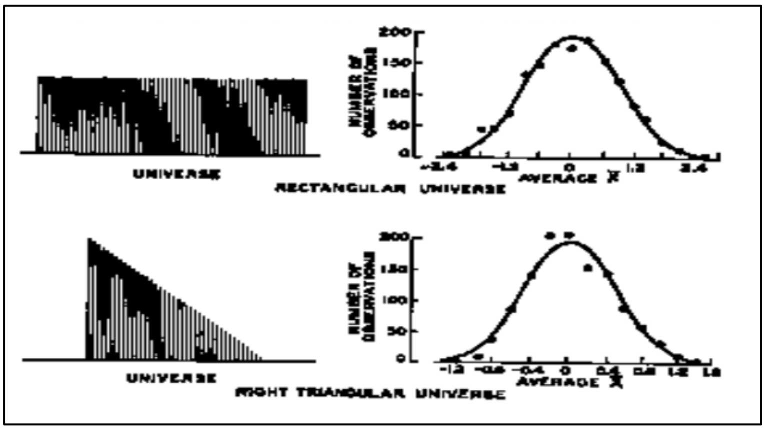

Figure 2.8 : Illustration of the central limit theorem.

Source : AJ sup and Watson, 1993

The practical implications of the central limit theorem are immense. Consider that without the central limit theorem effects, we would have to develop a separate statistical model for every non-normal distribution encountered in practice. This would be the only way to determine if the system were exhibiting chance variation. Because of the central limit theorem we can use averages of small samples to evaluate any process using the normal distribution. The central limit theorem is the basis for the most powerful of statistical process control tools, Shewhart control charts.